University of Jeddah
Faculty of Engineering
Department of Electrical & Computer Engineering

Summer Semester – 2016
Electromagnetic Fields (ECE 308)

Lecture 4 – Electrostatics - I

Ankit Patel
majorankit@gmail.com
www.majorankit.wix.com/majorankit

Maxwell’s Equation:

The modern theory of electromagnetism is based on a set of four fundamental relations known as **Maxwell’s Equations**.

$$\nabla \cdot \mathbf{D} = \rho_v,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Here,

E is Electric Field Intensity and **D** is Flux Density are relates by : $\mathbf{D} = \varepsilon \mathbf{E}$, where ε is Electrical Permittivity

H is Magnetic Field Intensity and **B** is Flux Density are relates by: $\mathbf{B} = \mu \mathbf{H}$, where μ is Magnetic Permeability

ρ_v is the Electric Charge Density per unit volume

J is the Current Density per unit area

Maxwell’s Equation:

Maxwell’s equations hold in any material, including free space (vacuum). In general, all of the quantities in Maxwell’s equations may depends on spatial location and time t.

By formulating these equations, published in 1873, James Clerk Maxwell established the first unified theory of electricity and magnetism.

Maxwell’s equations deduced from experimental observations reported by Coulomb, Gauss, Ampere, Faraday, and many others, not only encapsulate the connection between the electric field and electric charge and between the magnetic field and electric current, but also capture the bilateral coupling between electric and magnetic fields and fluxes.

Maxwell's Equation:

Under static conditions, none of the quantities appearing in Maxwell's equations are functions of time (i.e. ∂/∂t = 0). "This happens when all charges are permanently fixed in space, or, if they move, they do so at a steady rate so that $\boldsymbol{\rho}_v$ and **J** are constant in time." Under these conditions, Maxwell's equations rewrite as:

Electrostatics

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \times \mathbf{E} = 0.$$

Magnetostatics

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

This shows that Maxwell's equations separate into two uncoupled pairs, with the first pair involving only the electric field and flux **E** and **D**, while the second pair containing only the magnetic field and flux **H** and **B**.

Charge & Current Distributions:

In electromagnetics, we encounter various forms of electric charge distributions. When put in motion, these charge distributions constitute current distributions. Charges and currents may be distributed over a volume of space, across a surface, or along a line.

charge densities:

At the atomic scale, the charge distribution in a material is discrete, meaning that charge exists only where electrons and nuclei are and nowhere else. In electromagnetics, we can disregard the discontinuous nature of charge distribution and treat the net charge contained in an elemental volume Δv as if it were uniformly distributed within. Accordingly, we define the ***Volume Charge Density*** ρ_v as,

$$\rho_v = \lim_{\Delta \mathcal{V} \to 0} \frac{\Delta q}{\Delta \mathcal{V}} = \frac{dq}{d\mathcal{V}} \qquad (\mathrm{C/m^3})$$

Charge & Current Distributions:

charge densities:

In general , ρ_v depends on spatial location (x, y, z) and t; thus, $\rho_v = \rho_v(x, y, z, t)$. Physically, ρ_v represents the average charge per unit volume for a volume Δ*v* centered at (x, y, z), with Δ*v* being large enough to contain a large number of atoms, yet small enough to be regarded as a point at the macroscopic scale under consideration. The variation of ρ_v with spatial location is called its **Spatial Distribution**, or simply its **Distribution**. The total charge contained in volume *v* is,

$$Q = \int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$

Thank you !